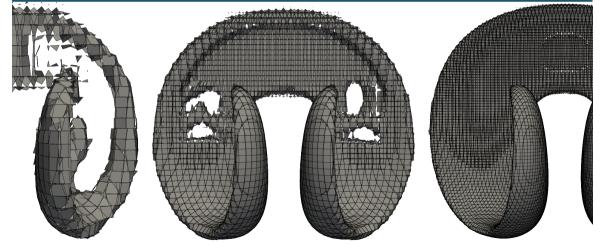
Software Design Patterns in Research Software with examples from OpenFOAM



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Software Design Patterns in Research Software with examples from OpenFOAM - Dr.-Ing. Tomislav Maric

Disclaimer

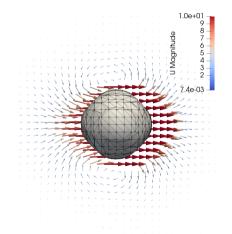


This webinar is about Design Patterns in Research Software, and I'll be using examples from my own work with OpenFOAM, a GPL open-source, but trademarked software:

This content is not approved or endorsed by OpenCFD Limited, producer and distributor of the OpenFOAM software via www.openfoam.com, and owner of the OPENFOAM® and OpenCFD® trademarks.

Research Software Direct Numerical Simulation methods of multiphase flows





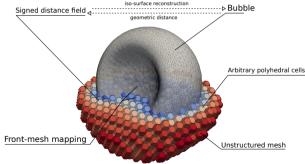
- Fluid phases that do not mix are separated by sharp interfaces (3D surfaces).
- Fluid phases exchange mass, momentum, and energy at fluid interfaces.
- Fluid interfaces deform, break up, and merge.
- Direct Numerical Simulations aim to resolve all scales, while ensuring convergence, volume conservation and (parallel) computational efficiency.

Multiphase flows are everywhere

 Fuel-cells, Lab-On-a-Chip, ship/offshore hydrodynamics, coating processes, 3D printing, ...

Research Software Unstructured Level Set / Front Tracking Method I



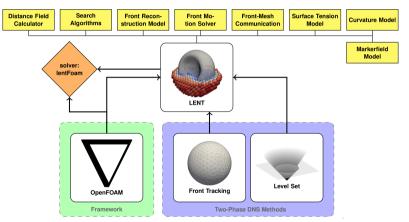


Level Set / Front Tracking [1, 2, 3, 4] on unstructured meshes [5, 6, 7, 8] **combines**

- Phase-indication (marker field): which fluid phase occupies point x at time t?
- Signed-distance calculation (redistancing): curvature approximation.
- Front (3D surface mesh) reconstruction: topology changes.
- Point-search operations: vertex-cell (front-mesh) mapping.
- Velocity interpolation.

Research Software Unstructured Level Set / Front Tracking Method II



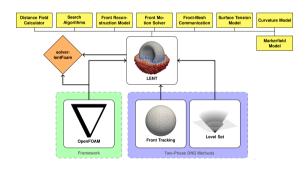


Research software development for LENT is done by Tobias Tolle 0, Jun Liu 0, and myself 0.

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Research Software Unstructured Level Set / Front Tracking Method III





The quality of the method is determined by validation & verification studies.

 There was a another IDEAS/ECP webinar (2021-04-07) that covers a workflow for increasing research software quality in this context.

The sub-algorithms build a hierarchy, whose elements should be interchangeable at runtime without changing existing code.

Object-Oriented Programming Crash Course I Encapsulation on one slide

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lentCommunication

// Triangle -> Cell : triangle vertex in cell.

-triangleToCell_: DynamicList<label>

// Vertex -> Cell : vertex in cell.

-vertexToCell_: DynamicList<label>

// Interface cell -> contained triangles (inverse of triangleToCell_) -interfaceCellToTriangles_: std::map<label, std::vector<label» // Interface cell -> contained vertices (inverse of vertexToCell)

-interfaceCellToVertices_: std::map<label, std::vector<label»

// Cell -> Nearest Triangle.

-cellsTriangleNearest_: DynamicList<pointIndexHit>

// Point -> Nearest Triangle.

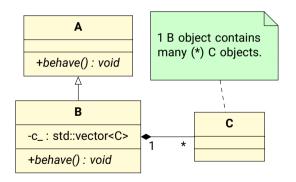
-pointsTriangleNearest_: DynamicList<pointIndexHit>

+update(): void +updateVertexToCell(): void +updateInterfaceCellToTriangles(): void +updateInterfaceCellToVertices() void Cannot talk about the hierarchy without understanding its elements first.

- Complex things (e.g. Front-Mesh communication) are abstracted in C++ as User-Defined Types (UDT, classes).
- A class encapsulates its data (attributes, data members).
- A class implements behavior: member functions that change the data members.
- Access specifiers
 - +: accessible from outside (public)
 - -: inaccessible from outside (private)
- Private data (-) = narrow focus.

Object-Oriented Programming Crash Course II Dynamic Polymorphism on one slide

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Cannot talk about the hierarchy without understanding the interactions between its elements (UML)

- Classes inherit (derive) from other classes: A inherits from B.
- Classes contain (composit) objects of other classes: A contains C.

Dynamic polymorphism: addressing an object of the derived class via a pointer to the base class can be used to set the type of the object at runtime.

```
configData input{"path/to/file"};
smart_pointer<A> Aptr = A::New(input);
Aptr->behave(); // B chosen in input!
```

Software Design Patterns in Research Software

What are software design patterns useful for?



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Support programming on a higher-level of abstraction

- A high-level of abstraction is crucial thinking in terms of complex objects; not getting lost in low-level details.
- Design patterns modularize abstractions' functionality and their interaction:
 - What do parcels require from the mesh in order to evolve?
 - Which objects are written with runTime.write()?

```
// Peform mesh changes
mesh.update()
```

```
// Update moving reference frame
MRF.update();
```

// Make the fluxes relative to the mesh-motion
fvc::makeRelative(phi, rho, U);

```
// Evolve the particle cloud
parcels.evolve();
```

```
// Evolve the surface film
surfaceFilm.evolve();
```

```
// Write data
runTime.write();
```

Software Design Patterns in Research Software Examples from OpenFOAM



Software Design Patterns [9]: code structures that **combine inheritance and composition** and have emerged repeatedly as **best-practice solutions for specific design problems**.

Software Design Patterns (examples from OpenFOAM)

- Template Method: boundary conditions, viscosity models, discretization schemes, ...
- Strategy: transport models, solvers and pre-conditioners, ...
- Observer: dynamic mesh handling, IO synchronization, ...
- OpenFOAM's Creational Pattern: Runtime-Type Selection (RTS), used everywhere.

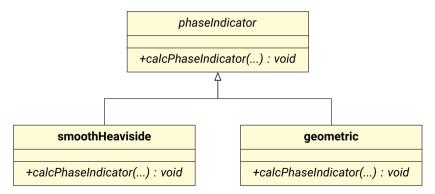
Not covered in this webinar

- Facade: Level Set / Front Tracking (Additional Slides)
- Curiously Recurring Template Pattern (CRTP): Discrete Parcel Method (Additional Slides)

Software Design Patterns in Research Software Template Method I



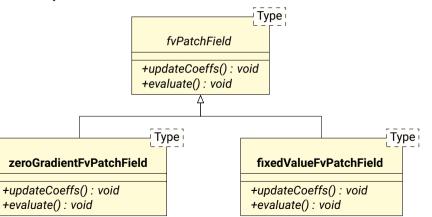
Virtual member function: implements different behavior in a derived class.



Software Design Patterns in Research Software Template Method II



OpenFOAM's boundary conditions

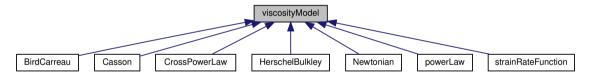


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Software Design Patterns in Research Software Template Method III



Viscosity model hierarchy



and the nu Template Method

// Return the laminar viscosity.
virtual tmp<volScalarField> nu() const = 0;

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Software Design Patterns in Research Software Template Method IV

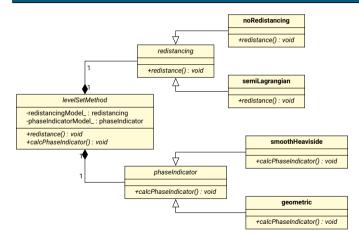


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- The Template Method is the virtual member function (method) to be overridden, it has nothing to do with C++ templates.
- Best practice: utilize virtual member functions (dynamic polymorphism) to extend existing libraries without modifying them.

Software Design Patterns in Research Software Strategy I

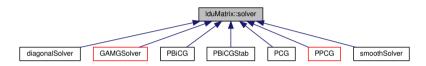




- A single class contains different sub-algorithms.
- Sub-algorithms can be selected at runtime.
- Combining sub-algorithms does not require programming.
- Basically the composition of the Template Method for sub-algorithm hierarchies.
- Best practice: when unsure about sub-algorithm combinations, implement the Strategy Pattern.

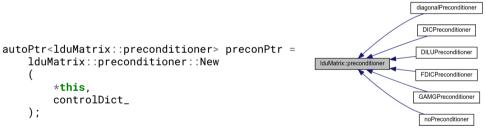
Software Design Patterns in Research Software Strategy II





```
Foam::lduMatrix
```

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Each lduMatrix::solver selects its pre-conditioner as a (preconditioning) Strategy.

Software Design Patterns in Research Software

Strategy III

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Software Design Patterns in Research Software Observer I



From GoF Design Patterns Book [9]: "Define a one-to-many dependency between objects so that when one object changes state, all its dependents are notified and updated automatically." **Subject**

- Has a state that is updated when the subject is modified.
- Forwards the **update** call to a list of its observers.

```
void subject::update()
{
   for (auto& observer : observers_)
        observer.update();
}
```

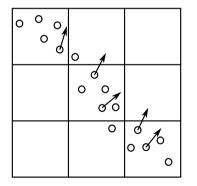
Observers

- Implement the update interface.
- Register themselves to the subject via their constructor.

Software Design Patterns in Research Software Observer II



Example: Particles tracked along Lagrangian trajectories in an Eulerian (background) mesh

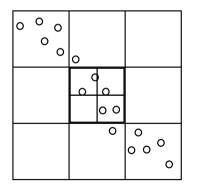


- Lagrangian-cloud particles know which cell they are in.
- The Eulerian mesh is the subject that changes state.
- Lagrangian particle cloud is <u>an</u> observer.
- Vice-versa is also relevant, resulting in 6-way coupling (mass, momentum, energy exchange × 2).

Software Design Patterns in Research Software Observer III



Example: Particles tracked along Lagrangian trajectories in an Eulerian (background) mesh



- 1. The Eulerian mesh (subject) changes state: it is refined.
- 2. The Eulerian mesh (subject) updates its observers for (auto& observer : observers) observer.update(cellMap);
- 3. The Lagrangian cloud is an observer

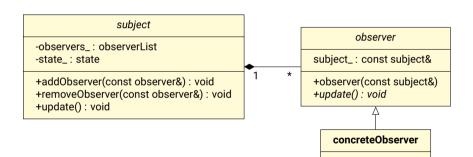
```
for (auto& particle : cloud)
```

```
auto found = cellMap.find(particle.cellLabel());
if (found)
```

```
auto newCellLabel = cloud.find(particle, cellMap);
particle.setCellLabel(newCellLabel);
```

Software Design Patterns in Research Software Observer IV



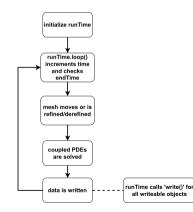


+update(): void

Software Design Patterns in Research Software Observer V



Example: write all data that should be written using the same output frequency

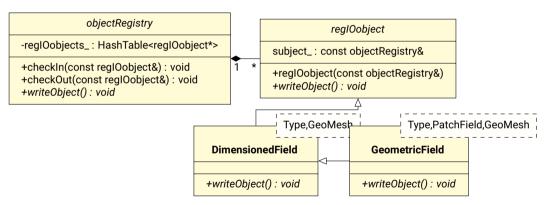


```
A sinale
runTime.write():
call in the solver application, and
for (regIOobject& : regIOobjects)
    regIOobject.writeObject();
in the Time class is better than manually typing
if (runTime.writeTime())
    alpha.write();
    surfaceMesh.write();
    cloud.write();
    . . .
```

in a solver application. It is necessary for reactive flows.

Software Design Patterns in Research Software Observer VI





Foam::Time controls simulation (write) time and it is an objectRegistry. Foam::Time::write() loops over all registered fields and writes them to the drive.

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Software Design Patterns in Research Software Observer VII



while (runTime.loop()) // runTime state

```
#include "CourantNo.H"
```

```
// Pressure-velocity PISO corrector
```

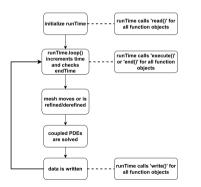
```
#include "UEqn.H"
// --- PISO loop
while (piso.correct())
{
    #include "pEqn.H"
}
```

```
laminarTransport.correct();
turbulence->correct();
runTime.write(); // runTime state
```

- A CFD solver is a procedural application.
- Fields (velocity, pressure, density, temperature, ...) are global variables, modified by FVM differential operators / solution algorithms.
- Observer Pattern simplifies custom post-processing using OpenFOAM Function Objects (not C++ function objects).

Software Design Patterns in Research Software Observer VIII



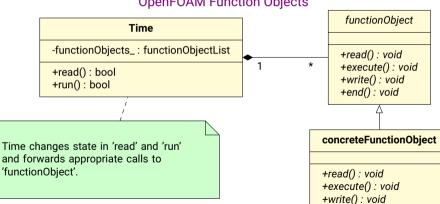


- runTime is the subject that changes state:
 - time-step increment
 - reached output time
 - reached end time
- **Function Objects** are the **observers**.
 - They access other (mesh or time) observers and "work" on them: compute the maximal and minimal temperature, sample the veloity over a line segment, ...
- OpenFOAM Function Objects change solver behavior without modifying solver application's source code.

Software Design Patterns in Research Software Observer IX



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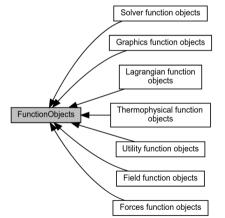
OpenFOAM Function Objects

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+end(): void

Software Design Patterns in Research Software Observer X



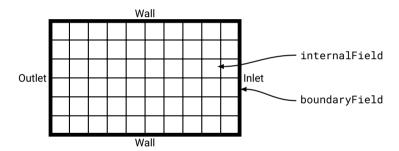


OpenFOAM Function Objects

- Observer is also used within Function Objects themselves: fvMesh is an objectRegistry, FOs fetch objects registered to the mesh and perform live (post-)processing tasks as the simulation runs.
- This saves research time and HPC resources (green computing): live post-processing can be used to stop large-scale simulations as soon as the results are too erroneous.

Software Design Patterns in Research Software Observer XI



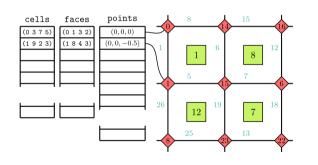


Geometric Fields:

- Values grouped into internal values and boundary patch values.
- Internal values associated with cell centers (alternatively: face centers or cell corner-points), boundary with face centers (alternatively, face corner-points).

Software Design Patterns in Research Software Observer XII





- The mesh connectivity changes with mesh refinement / unrefinement.
 - GeometricFields do not map to the mesh.
- Mesh motion stretches/compresses finite volume faces.
 - Volumetric fluxes change magnitudes.
- Each time the mesh is **updated**, the fields are **updated**.
- fvMesh is a Subject,
 GeometricFields are the Observers.

Software Design Patterns in Research Software Observer XIII



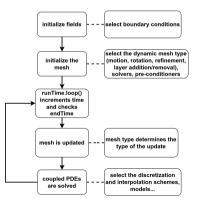
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Best practice

Use when the same member function (write, map, execute, read, update) must be called for many objects.

Software Design Patterns in Research Software Runtime Type-Selection I





- Using a Creational Pattern to construct objects (select types) at runtime makes the solver application highly configurable.
- No modification to the solver application is required to select boundary conditions, dynamic mesh handling, discretization and interpolation schemes, models, ...

Software Design Patterns in Research Software Runtime Type-Selection II



Runtime Type Selection (RTS) is OpenFOAM's Creational Pattern.

RTS constructs OpenFOAM objects based on user input.

- Ease-of-use: RTS tables provide information about available types and their parameters.
- Simplifies research: "constructing" the PDE discretization and solution via configuration files.

• OpenFOAM's RTS in a nutshell:

- RTS stores a class-static hash-table that maps strings to a virtual member function pointer.
- Description This so-called RTS table is initialized for the base class in its implementation file.
- The RTS table is extended in implementation files of derived classes.
- The RTS code is generated using preprocessor macros
 - RTS declaration and definition
 - RTS table extension

Software Design Patterns in Research Software Runtime Type-Selection III



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 Best practice: if a research software provides a creational pattern, learning how to use it simplifies testing and saves time in research, compared to hacking your own "if-then-else" code for different types.

Software Design Patterns in Research Software

Runtime Type-Selection IV



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OpenFOAM RTS macros expanded with gcc -E: no need to learn how this works to use it

```
typedef autoPtr<implicitSurface> (*ITstreamConstructorPtr)( ITstream is ):
    typedef ::Foam::HashTable <TTstreamConstructorPtr. ::Foam::word, ::Foam::Hash<::Foam::word> > TTstreamConstructorTableType:
    typedef ::Foam::HashTable < std::pair<::Foam::word.int>. ::Foam::Word. ::Foam::Hash<::Foam::word> > ITstreamConstructorCompatTableType:
    static ITstreamConstructorTableType* ITstreamConstructorTablePtr :
    static std::unique ptr<ITstreamConstructorCompatTableType> ITstreamConstructorCompatTablePtr :
    static ITstreamConstructorCompatTableType& ITstreamConstructorCompatTable():
    static void ITstreamConstructorTablePtr construct(bool load):
    static ITstreamConstructorPtr ITstreamConstructorTable(const ::Foam::word& k):
    template<class implicitSurfaceType> struct addAliasITstreamConstructorToTable {
    explicit addAliasITstreamConstructorToTable ( const ::Foam::word& k, const ::Foam::word& alias, const int ver ) {
        ITstreamConstructorCompatTable() .set(alias. std::pair<::Foam::word.int>(k.ver)):
    template<class implicitSurfaceType> struct addITstreamConstructorToTable {
        static autoPtr<implicitSurface> New ( ITstream is ) {
            return autoPtr<implicitSurface>(new implicitSurfaceType (is)):
        } explicit addITstreamConstructorToTable ( const ::Foam::word& k = implicitSurfaceType::typeName ) {
            ITstreamConstructorTablePtr construct(true):
            if (!ITstreamConstructorTablePtr ->insert(k, New)) {
                std::cerr << "Duplicate entry " << k << " in runtime table " << "implicitSurface" << std::endl:</pre>
                ::Foam::error::safePrintStack(std::cerr):
        } ~addITstreamConstructorToTable() {
            ITstreamConstructorTablePtr construct(false):
        } addITstreamConstructorToTable (const addITstreamConstructorToTable&) = delete:
        void operator= (const addITstreamConstructorToTable&) = delete: }:
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                                                                                                          2022-03-08 IDEAS Productivity | ECP
```

Software Design Patterns in Research Software



Software Design Patterns [9]: design structures that **combine inheritance and composition** and have emerged repeatedly as **best-practice solutions for specific design problems**.

Software Design Patterns (examples from OpenFOAM)

- Template Method: boundary conditions, viscosity models, discretization schemes, ... ✓
- Strategy: transport models, solvers and preconditioners, ... \checkmark
- Observer: dynamic mesh handling, IO synchronisation
- OpenFOAM's Creational Pattern: Runtime-Type Selection (RTS), used everywhere. \checkmark

Not covered in this webinar

- Facade: Level Set / Front Tracking (Additional Slides)
- Curiously Recurring Template Pattern (CRTP): Discrete Parcel Method (Additional Slides)

Software Design Patterns in Research Software Traits + RTS + Template Method = Domain-Speific Language for PDEs



```
fvScalarMatrix TEqn
(
    fvm::ddt(T)
    + fvm::div(phi, T)
    - fvm::laplacian(DT, T)
    ==
    fvOptions(T)
);
TEqn.solve();
```

We didn't cover everything, but

 Type Lifting for geometric fields and differential operators "+" generic traits for tensor rank calculation "+" Template Method and RTS for discretization and interpolation schemes + Strategy and RTS for linear solvers "=" OpenFOAM's Domain-Specific Language for Partial Differential Equation discretization.

Software Design Patterns in Research Software Conclusions / Discussion



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- Design Patterns speed up research, if there is a high degree of methodological uncertainty: we don't know which algorithms will work, in which combination.
- Avoiding dogmatism: not every design question has to be answered by a pattern.
- When dealing with legacy research code, it helps a lot understand its design principles: cargo-cult programming is quicker, but can tank research projects in the long-run.

Software Design Patterns in Research Software Acknowledgements



Funded by the German Research Foundation (DFG) - Project-ID 265191195 - CRC 1194



Interaction between Transport and Wetting Processes

Z-INF sub-project (Prof. Dr. rer. nat. Dieter Bothe O, Prof. Dr. Christian Bischof O)

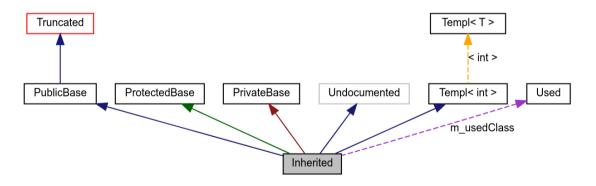
Additional Slides and References

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Object-Oriented Programming Crash Course III UML in OpenFOAM



OpenFOAM's UML legend



C++ Generic Programming in OpenFOAM (crash course) I Useful techniques



OpenFOAM uses Generic Programming (GP) for type lifting and traits.

- Type lifting: same code is re-used without modification with completely unrelated types. In OpenFOAM, everything is type-lifted for all tensors (scalar, vector, tensor, symmetric tensor, spherical tensor).
- Template specialization: e.g. specializing a fixed value tensor boundary condition as a scalar total pressure boundary condition.
- **Traits:** determine the tensor rank of the return type of $\nabla \mathbf{v}$ (used in differential operators).

C++ Generic Programming in OpenFOAM (crash course) II C++ templates: if-then-else for types



```
template<class Type>
Type sum(const UList<Type>& f)
{
    if (f.size())
    {
        Type Sum = pTraits<Type>::zero;
        TFOR_ALL_S_OP_F(Type, Sum, +=, Type, f)
        return Sum;
    }
    else
    {
        return pTraits<Type>::zero;
    }
}
```

template(Merriam Webster dictionary)

- a gauge, pattern, or mold (such as a thin plate or board) used as a guide to the form of a piece being made
- a molecule (as of DNA) that serves as a pattern for the generation of another macromolecule (such as messenger RNA)
- something that establishes or serves as a pattern

C++ Generic Programming in OpenFOAM (crash course) III Type Lifting



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template<class Type>
class fixedValueFvPatchField

public fvPatchField<Type>

- A boundary condition class template is type-lifted all tensors.
- The same is done for arithmetic field operators, discretization schemes, ...

C++ Generic Programming in OpenFOAM (crash course) IV Template Specialization



#define makePatchTypeFieldTypedef(fieldType, type) \ typedef type##FvPatchField<fieldType> \ CAT4(type, FvPatch, CAPITALIZE(fieldType), Field);

class totalPressureFvPatchScalarField

// fixedValueFvPatchField<scalar> public fixedValueFvPatchScalarField

Specialized boundary conditions for pressure, temperature, velocity,...

C++ Generic Programming in OpenFOAM (crash course) V Traits I



```
template<class Type>
tmp
<
    GeometricField
    imp
    imp
```

- The return-type of the gradient function template is determined based on the argument.
- The gradient of a scalar field is a vector field.

C++ Generic Programming in OpenFOAM (crash course) VI Traits II



```
template<class arg1, class arg2>
class outerProduct
{
public:
```

```
typedef typename typeOfRank
{
    typename pTraits<arg1>::cmptType,
    direction(pTraits<arg1>::rank) + direction(pTraits<arg2>::rank)
    >::type type;
};
```

- Traits determine the component types of scalars, vectors, tensors.
- Component type and rank traits promote outer product type.
- One only needs this if the research involves extending the set of differential operators.
- Type-lifting is enough for 99% of research using OpenFOAM.

Combining Object-Oriented and Generic Programming



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template<class Type> class inletOutletFvPatchField

public mixedFvPatchField<Type>

protected:

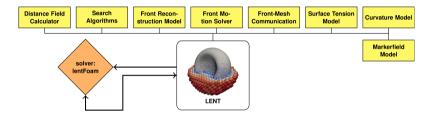
```
// Protected data
```

```
//- Name of flux field
word phiName_;
```

- OpenFOAM combines Generic and Object Oriented Programming.
- Makes sense: e.g. the inlet-outlet boundary condition is a mixed boundary condition, and it behaves exactly the same way for different tensors.
- Using OOP here for the tensor Type is much more cumbersome and potentially slower than using type lifting.

Software Design Patterns in Research Software Facade I





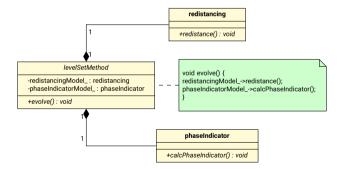
Facade hides the complexity of sub-algorithms, for example, the order of execution:

```
void lent::advect()
{
    frontReconstructionModel->reconstructFront(); // Updates Front-Mesh communication.
    frontMotionSolver->evolveFront(); // Front-Mesh Comm. update, using Search Algorithms.
    distanceFieldCalculator->calcSignedDistances();
}
```

Software Design Patterns in Research Software Facade II







Best practice: implement sub-algorithms as Strategies, test them individually, then integrate them in a specific execution order using Facade.

Software Design Patterns in Research Software Curiously Recurring Template Pattern I



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template<typename Parameter> class MyType

: public Parameter

- Class template inheriting from its template parameter.
- Used in generic programming for policy-based design: extending the host class (MyType) interface by inheriting from the template parameter (Parameter).

Software Design Patterns in Research Software Curiously Recurring Template Pattern II



Curiously Recurring Template Pattern (CRTP) is **couriously recurring and nested** for the Lagrangian / Eulerian Discrete Parcel Method.

```
template<class CloudType>
class ReactingCloud
```

```
public CloudType,
public reactingCloud
```

References I



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